Conditions Imposed by Process Statics on Multivariable Process Dynamics

The steady-state behavior of a multivariable process system imposes certain necessary conditions on its dynamic behavior, particularly in any feedback control system containing integral action. These conditions are derived in general and compared with previous restricted results. A more flexible design view is allowed by the general result.

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SCOPE

Models of the steady-state behavior of processes are generally more accurate and readily available than are those of the unsteady-state. Therefore, it is of interest to know in the general case how much design information about unsteady-state behavior can be obtained from a steady-state model. Since a steady-state model can be viewed as a simplification of an unsteady-state model, obtained by setting time derivatives to zero, there is reason to expect that the former contains at least some information about the latter.

Two previous works have derived results of this nature for linear control systems. Niederlinski (1971) derived a necessary and sufficient condition for the existence of a stable linear feedback control, consisting of an arbitrary number of individual single-input-single-output (SISO) stable feedback loops containing integral action, for processes with linear dynamics. Using this theorem and steady-state information only, certain pairings of input and output variables in the control loops can be eliminated as dynamically unstable for any controller tuning. Bristol (1966) derived a relative gain array (RGA) for similarly restricted systems. In the case of two loops, the RGA gives the identical result to Niederlinski's theorem. For control systems of higher dimension, the RGA gives qualitative guidelines on the relative desirability of alternative pairing of the input and output variables. Both results use only information about the steady-state behavior of the process to deduce attributes of the closed-loop unsteady-state behavior.

Most control systems contain nonlinear processes. And, although SISO control structures containing integral feedback action are prevalent in practice, other more sophisticated multivariable control strategies are being increasingly applied, particularly as computer control becomes more common. Neither nonlinearities nor more general multivariable control schemes are addressed by these previous studies.

Both the RGA and Niederlinski's theorem are used to guide the choice of input-output pairing in the multiloop SISO structure. They are particularly useful when they can unequivocally eliminate certain pairings. However, if the control structure is allowed to be more general, even in the case of linear systems it is not readily clear as to just what can be eliminated a priori by these results.

The RGA and Niederlinski's theorem are both closed-loop results. They use information about the open-loop process statics to deduce information about closed-loop stability. It is of interest to know whether any information about open-loop dynamics can be deduced from open-loop statics. Processes exhibiting multiple steady states appear to offer the most potential for yielding this type of result, because of the existence of general mathematical conditions describing the derivatives of the process statics in such cases.

In the present study, we remove the restrictions of linear processes and of individually stable, linear SISO loops, each with integral action. The objective is to derive results comparable to Niederlinski's and Bristol's results for these more general circumstances. In addition, we look specifically at systems exhibiting multiple steady states. The objective of this part of the study is to determine whether the special nature of the process statics in these circumstances allows more detailed inference about the process dynamics. Attention is restricted to two cases of SISO systems: closed-loop systems with input multiplicities, and open-loop systems with output multiplicities.

CONCLUSIONS AND SIGNIFICANCE

For arbitrary multivariable systems, using any form of feedback control containing integral action, the generalized form of necessary condition for the possibility of closed-loop stability is given by Eq. 22. This result specializes to those of Niederlinski and Bristol when their additional restrictions are imposed. When used in an unrestricted framework, as in the design of a sophisticated multivariable control system for a nonlinear process, Eq. 22 shows that only the structure of any integral feedback is restricted by the process steady-state characteristics. Equation 22 quantifies this restriction in general terms for any process dynamics and for any feedback control structure.

Even if a simple control strategy is to be designed, Eq. 22 permits a more flexible design viewpoint. For example, for the dynamically most important parts of the control system, the process illustrated by Eq. 30 can in fact be viewed as it is shown in Eq. 31. Only the integral feedback, important for eliminating steady-state offset but relatively unimportant for dynamic effects, must be viewed as in Eq. 30.

For SISO systems having input multiplicities, as illustrated by Figure 1, Eq. 22 can be used to show that only steady states having the same sign of process gain can be closed-loop stable for a particular tuning of any controller containing integral feedback. Adjacent steady states cannot both be closed-loop stable.

For SISO systems with output multiplicities, the methods of this work, particularly Eq. 19, explain why in typical examples, such as the CSTR, adjacent steady states are not both open-loop stable.

DERIVATION OF THE CLOSED-LOOP NECESSARY CONDITION

We assume the following standard general form of process dynamics

$$\frac{dx}{dt} = g(x,u) \tag{1}$$

$$y = f(x) \tag{2}$$

where $x(t)$ is a vector of p state variables; $u(t)$ is a vector of n-manipulated inputs; $y(t)$ is a vector of n-controlled outputs; and, g and f are vectors of p and n functions, respectively, g describing the state dynamics and f describing how the output variables relate to the state and input variables. We assume for now that the dynamic system represented by Eq. 1 is stable. The objective is to derive necessary conditions, depending only on steady-state process open-loop characteristics, for the control system to have the possibility of closed-loop stability under quite general feedback control.

The feedback control will be represented by the general form

$$u = h\left(e, \int_0^t e\,dt\right) \tag{3}$$

The function h is a vector of n feedback relations, and the error e a vector of n discrepancy functions

$$e = r - y \tag{4}$$

where r is a vector of n setpoint values. This control equation represents any form of multivariable feedback control, with reset (integral) action included to eliminate offset.

We augment the state vector by defining $z = \int_0^t e\,dt$, so that

$$\frac{dz}{dt} = e \tag{5}$$

and Eq. 3 becomes

$$u = h(e,z) \tag{6}$$

Equation 5 serves the usual purpose of integral action. It prevents the control system from reaching any steady state which does not satisfy $e = 0$, thereby eliminating offset. Equation 3 can obviously be specialized to represent commonly occurring schemes, such as multiloop proportional-plus-integral control.

Equations 1, 2, 4, 5 and 6 describe the control system dynamics. At steady state they must satisfy:

$$\begin{aligned} g(x_s,u_s) &= 0 \\ y_s = f(x_s) &= r \\ e_s &= 0 \\ u_s &= h(0,z_s) \end{aligned} \tag{7}$$

where subscript s indicates a steady-state value. We linearize these equations around the steady state described by Eqs. 7, to obtain after algebraic rearrangement

$$\begin{aligned} \frac{dX}{dt} &= \left(\frac{\partial g}{\partial x} - \frac{\partial g}{\partial u}\frac{\partial h}{\partial e}\frac{\partial f}{\partial x}\right)X + \frac{\partial g}{\partial u}\frac{\partial h}{\partial z}Z \\ \frac{dZ}{dt} &= \frac{\partial f}{\partial x}X \end{aligned} \tag{8}$$

where the deviation variables are defined as

$$\begin{aligned} X &= x - x_s \\ Z &= z - z_s \end{aligned} \tag{9}$$

Equations 8 are a system of $(n + p)$ linear differential equations, which we can write in the combined form

$$\frac{d\begin{pmatrix} Z \\ X \end{pmatrix}}{dt} = A\begin{pmatrix} Z \\ X \end{pmatrix} \tag{10}$$

where the partitioned $[(n + p) \times (n + p)]$ square matrix A is given by:

$$A = \begin{pmatrix} 0 & -\frac{\partial f}{\partial x} \\ \frac{\partial g}{\partial u}\frac{\partial h}{\partial z} & \frac{\partial g}{\partial x} - \frac{\partial g}{\partial u}\frac{\partial h}{\partial e}\frac{\partial f}{\partial x} \end{pmatrix} \tag{11}$$

By forming the general characteristic equation $|A - \lambda I| = 0$, it is easy to show (Fadeeva, 1959) that a necessary condition for stability of Eq. 10, i.e., a necessary condition for all eigenvalues of A to have negative real parts, is

$$sgn\,|A| = (-1)^{n+p} \tag{12}$$

where the sgn function is defined by

$$sgn\,x = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$$

Therefore, we evaluate $|A|$. To do this we will employ the following identity for the determinant of a partitioned square matrix:

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = |D| \cdot |A - BD^{-1}C| \tag{13}$$

which holds provided A and D are also square, and provided D^{-1} exists. This identity is proved by Rickard (1983).

Assuming for now that $\partial h/\partial z$ is not singular, define the partitioned $[(n + p) \times (n + p)]$ square matrix E as

$$E = \begin{pmatrix} I & \left(\frac{\partial h}{\partial z}\right)^{-1}\frac{\partial h}{\partial e}\frac{\partial f}{\partial x} \\ 0 & I \end{pmatrix} \tag{14}$$

and form the product AE

$$AE = \begin{pmatrix} 0 & -\frac{\partial f}{\partial x} \\ \frac{\partial g}{\partial u}\frac{\partial h}{\partial z} & \frac{\partial g}{\partial x} \end{pmatrix} \tag{15}$$

By Eq. 13

$$|AE| = \left|\frac{\partial g}{\partial x}\right| \cdot \left|\frac{\partial f}{\partial x}\left(\frac{\partial g}{\partial x}\right)^{-1}\frac{\partial g}{\partial u}\frac{\partial h}{\partial z}\right| \tag{16}$$

But, since Eq. 13 shows that $|E| = 1$, Eq. 16 gives the value of $|A|$, as desired for use of Eq. 12.

Let the steady state input-output behavior of Eqs. 1 and 2 be represented by $c(m)$, where c is a steady-state value of output y which will satisfy Eqs. 1 and 2 for the steady-state value m of u:

$$g(x,m) = 0$$
$$c = f(x) \tag{17}$$

By linearizing and eliminating x we can get the linearized input-output dependence of c on m at a steady state. Thus

$$\frac{\partial g}{\partial x} dx + \frac{\partial g}{\partial u} dm = 0$$
$$dc = \frac{\partial f}{\partial x} dx \tag{18}$$

yields

$$\frac{\partial c}{\partial m} = -\frac{\partial f}{\partial x}\left(\frac{\partial g}{\partial x}\right)^{-1}\frac{\partial g}{\partial u} \tag{19}$$

This identifies one of the groupings in Eq. 16 with the steady-state gain matrix $\partial c/\partial m$.

The linearized open-loop dynamics are represented by Eq. 8 with $h = 0$, or

$$\frac{\partial X}{\partial t} = \frac{\partial g}{\partial x} X \tag{20}$$

Restricting attention to systems which are open-loop stable, by the same reasoning which led to Eq. 12 we conclude

$$sgn\left|\frac{\partial g}{\partial x}\right| = (-1)^p \tag{21}$$

Combining Eqs. 12, 16, 19 and 21, we derive the general necessary condition for stability as:

$$sgn\left(\left|\frac{\partial c}{\partial m}\right| \cdot \left|-\frac{\partial h}{\partial z}\right|\right) = (-1)^n \tag{22}$$

This condition depends only on the steady-state gain matrix, and the strategy chosen for use of integral feedback action. Equation 22 will be shown below to be a generalized version of the necessity part of Niederlinski's theorem.

The assumption of nonsingular $\partial h/\partial z$ can be examined through Eq. 22. A nearly singular control strategy for the reset action will result in a borderline situation regarding the necessary stability condition. Slight changes in $\partial c/\partial m$, inevitable in a real operating control system, are thus likely to destabilize the system. Therefore, we can eliminate as impractical the choice of a control strategy with a singular $\partial h/\partial z$.

The general necessary condition in Eq. 22 shows that the process steady-state characteristics, i.e., the gain matrix $\partial c/\partial m$, impose a restriction on the choice of integral feedback structure $\partial h/\partial z$ in order to have the possibility of stable closed-loop behavior. We next examine previous results for less general systems to illustrate the implications of Eq. 22 for control system design practice.

COMPARISON WITH PREVIOUS RESULTS

The results of Niederlinksi (1971) and Bristol (1966) both yield related necessary conditions for less general systems. A major implication of Eq. 22 is that only the integral (reset) feedback action is really involved in these previous necessary conditions. After a brief discussion relating Eq. 22 to these previous results, we explore this in observation in more detail.

Niederlinski (1971) derived a necessary and sufficient condition for $n \times n$ multivariable control systems. His work imposed these restrictions: (1) the process dynamics are linear; (2) the control system is made up of n single-loop linear feedback controllers, each containing integrating action; (3) each individual closed loop is stable; and (4) each individual open loop is stable. Equation 22 requires *only* a generalized version of the fourth restriction for its validity.

To apply Niederlinski's restrictions to Eq. 22, we first define a sign-normalized version $\partial c^+/\partial m$ of the steady-state gain matrix $\partial c/\partial m$. It is derived from $\partial c/\partial m$ by multiplying columns or rows by (-1) as needed to produce a positive diagonal,

$$\frac{\partial c_j^+}{\partial m_j} > 0 \qquad j = 1, 2, \ldots n \tag{23}$$

The sign-normalized $\partial c^+/\partial m$ is not unique. For example, if

$$\frac{\partial c}{\partial m} = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$$

then

$$\frac{\partial c^+}{\partial m} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \text{ or } \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

depending on whether one chooses to multiply the second row or second column by (-1). The physical relation of $\partial c^+/\partial m$ to $\partial c/\partial m$ is simple. Multiplication of the jth column by (-1) corresponds to redefining the jth component of m by $m'_j i = -m_{ji}$ this occurs if the jth control valve is changed from air-to-open to air-to-close, or vice versa. Similarly, multiplication of the ith row by (-1) corresponds to redefining $c'_j = -c_i$, which occurs if the output action of the ith transmitter is reversed. Either of these changes is accommodated by appropriate selection of the direction of the corresponding controller output action to satisfy Niederlinski's restriction (3), and can have no effect on the inherent stability of the system.

Using $\partial c^+/\partial m$, we can incorporate the second and third restriction by writing a diagonal control strategy for the reset action

$$\frac{\partial h}{\partial z} = K_R \tag{24}$$

where K_R is an $n \times n$ diagonal matrix of n reset rates k_{R_i}, with $k_{R_i} > 0$ for $i = 1, 2, \ldots, n$. Then, since sgn $|-K_R| = (-1)^n$, Eq. 22 becomes

$$\left|\frac{\partial c^+}{\partial m}\right| > 0 \tag{25}$$

This compact form is identical to Niederlinski's theorem for multiloop controller pairing. However, since we have not assumed Niederlinski's restriction of linear process dynamics, Eq. 22 is a necessary condition only, while Niederlinski's theorem is both necessary and sufficient.

The relative gain array (RGA) devised by Bristol (1966) is one of the most useful design tools for pairing inputs and outputs in multiloop control systems. Recent examples of its use are McAvoy (1981) and Tung and Edgar (1981); these authors also cite several other papers using the RGA. Among the key advantages of Bristol's RGA is the fact that it requires information only about the steady-state behavior of the process, and not the dynamics. Bristol stated properties of the RGA, and presented 2 × 2 examples to give several guidelines for use of the RGA to address the multivariable pairing problem. The RGA is derived for systems satisfying restrictions similar to those stated for Niederlinski's results. For systems of dimension 2 × 2, the guidelines given by Bristol are equivalent to Niederlinski's necessary condition.

In studying processes of higher dimension than 2 × 2, it is not difficult to generate examples in which Bristol's guidelines must be used flexibly to reach useful conclusions. For example, the 3 × 3 process with steady-state gain matrix

$$\frac{\partial c^+}{\partial m} = \begin{bmatrix} \frac{5}{3} & 1 & 1 \\ 1 & \frac{1}{3} & 1 \\ 1 & 1 & \frac{1}{3} \end{bmatrix} \tag{26}$$

represents a non-allowable pairing by Eq. 25, since $|\partial c^+/\partial m| = -4/27$. Bristol's relative gain array μ for this case is

$$\mu = \begin{pmatrix} 10 & -4.5 & -4.5 \\ -4.5 & 1 & 4.5 \\ -4.5 & 4.5 & 1 \end{pmatrix} \qquad (27)$$

Since all diagonal elements are positive, the RGA guidelines cannot eliminate this pairing, even though it is not allowable.

While Niederlinski's result deals only with process statics, Bristol's RGA is capable of extension to consider process dynamics, as is done in the papers by McAvoy(1981) and Tung and Edgar (1981). On the other hand, as shown in the above example, Niederlinski's result is capable of unequivocal elimination of pairings not eliminated by the RGA. These considerations suggest that the two results be used in tandem.

Both Niederlinski's and Bristol's results are helpful in selecting input-output pairs for multivariable process control systems. Equation 22 allows a more flexible design interpretation of these results, as will be discussed next.

STRUCTURE OF MULTIVARIABLE CONTROL SYSTEMS

In practice, there is a predominance of single-input-single-output (SISO) loops with proportional-integral (-derivative) control action as a control strategy for multivariable process systems. Both Niederlinski's result, Eq. 25, and Bristol's 2×2 RGA have generally been viewed as necessary conditions for input-output pairing in these systems, e.g., for choosing which control valve should be used to regulate which measured variable.

The result of this paper, Eq. 22, emphasizes that only the integral action is really involved in this necessary condition for pairing, and shows how the condition is extended to more general multivariable control strategies.

For example, optimal control strategies normally do not include integral action, since they are based on a dynamic model which presumably represents the process and eliminates the need for integral feedback to remove offset. In practice, to use these controllers it is necessary to add some integral (reset) feedback to the optimal control in order to eliminate the offset generated by the inevitable discrepancy between actual and model steady-state behavior. Suppose an optimal feedback control

$$u = h^*(e) \qquad (28)$$

is derived for the dynamic system represented by Eqs. 1, 2 and 4. Then, a possible practical control would be

$$u = h\left(e, \int e\,dt\right) = h^*(e) + K_R \int_0^t e\,dt \qquad (29)$$

where K_R is as defined for Eq. 24. The general necessary condition, Eq. 22, imposes restrictions only on the reset pairing implied in K_R, and not on any pairing implied by the optimal control $h^*(e)$. One can thus use the optimal structure directly for the main control action, without regard to any "pairing" considerations, which affect only the parallel integral trim control action.

As another example, consider the 2×2 linear process

$$G_p(s) = \begin{pmatrix} \dfrac{2e^{-10s}}{s+1} & \dfrac{1}{s+1} \\ \dfrac{1}{s+1} & \dfrac{2e^{-10s}}{s+1} \end{pmatrix} \qquad (30)$$

For SISO control with two proportional-integral (-derivative) loops, it is generally assumed that only the pairing implied by Eq. 30 can be used, since $|\partial c^+/\partial m| < 0$ for the opposite pairing

$$G_p(s) = \begin{pmatrix} \dfrac{1}{s+1} & \dfrac{2e^{-10s}}{s+1} \\ \dfrac{2e^{-10s}}{s+1} & \dfrac{1}{s+1} \end{pmatrix} \qquad (31)$$

This pairing, while more attractive from a dynamic standpoint, is presumably eliminated by Niederlinski's result, Eq. 25, or by the RGA. However, Eq. 22 shows that only the structure of the integral feedback action is restricted by these necessary conditions. The proportional and dynamic lead actions, which form the major part of the dynamic control strategy, can be structured without regard to these conditions.

It remains to be investigated whether practical advantage can be gained from this recognition that the necessary conditions restrict only the structure used for integral feedback action.

SISO SYSTEMS WITH MULTIPLE STEADY-STATES

In this section, we restrict attention to systems having one manipulated variable and one controlled output, so that u, m, y and c are scalar functions of time, f is a scalar function of the state vector x, and h is a scalar function of the scalar error and its integral. It should be noted that, in multivariable systems made up of a collection of SISO loops, each loop can be regarded this way, with all the other loops closed. In addition, the systems considered will have multiple steady states. The objective is to determine what dynamic information can be obtained from the static characteristic, in two different situations: closed-loop systems with input multiplicity; and, open-loop systems with output multiplicity.

Closed-Loop Systems with Input Multiplicity

The steady-state characteristic is illustrated by Figure 1. In this particular case, there are three possible input-multiple steady-states yielding an output value $c = 0$, any one of which could conceivably be found by a control system containing integral feedback action (Koppel, 1982).

The general result, Eq. 22, specializes to

$$\frac{\partial c}{\partial m} \cdot \frac{\partial h}{\partial z} > 0 \qquad (32)$$

for SISO systems. This simply says that the product of process gain and integral feedback gain must be positive at any closed-loop steady-state which is stable. Note that there is no such restriction on the proportional gain. For example, the linear process $G(s) = 1/(s+1)$ with a steady state at $c = m = 0$, can be controlled stably at this steady state by any proportional controller with gain exceeding -1.

With reference to Figure 1, Eq. 32 shows that for a controller with a given sign of the integral feedback action, a pair of steady-states having process gains with opposite signs cannot both be stable. If, for example, the controller has positive gain on the integral feedback action, only steady-states 1 and 3 can be stable; steady-state 2 must be unstable. These statements are true regardless of the process dynamics, and for any control strategy.

Without attempting a rigorous definition of adjacency, but using it as a descriptive term, we can paraphrase this result to say that two adjacent input-multiple steady states cannot both be closed-loop stable under the same control when reset feedback is included.

Open-Loop Systems with Output Multiplicity

If the multiplicity appears as in Figure 2, i.e., as an output multiplicity, then it is obvious that only one steady state can be stable if reset feedback control is included. It will be the steady state corresponding to the chosen setpoint value for c. No other steady-state will be allowed by the reset action. Thus, process statics impose no particular restrictions on the closed-loop stability of output-multiple steady states.

The question of restrictions imposed by the steady-state characteristic on the open-loop stability is not as readily resolved. Physical examples, such as the single exothermic CSTR, often exhibit behavior in which all steady states between points 4 and 5 (Figure 2), where dc/dm is infinite, are open-loop unstable, if the

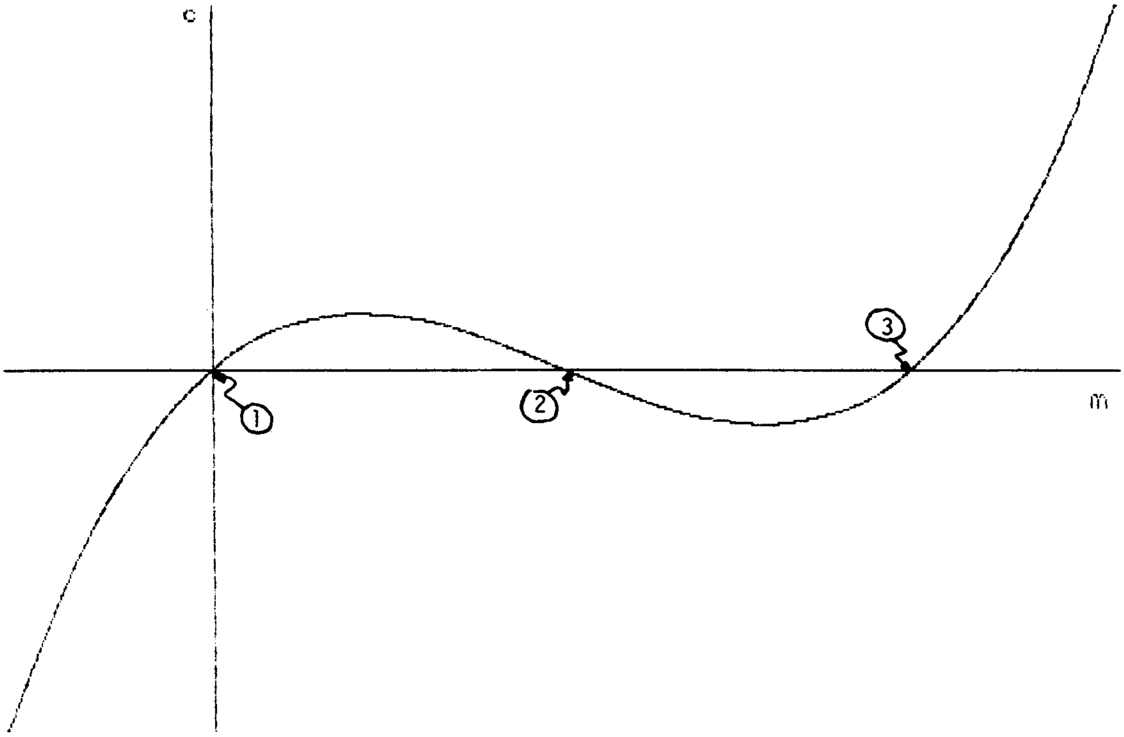

Figure 1. Steady-state characteristic of a process with input multiplicity.

remaining steady states are open-loop stable. The results of Kubicek et al. (1980) illustrate this behavior, which appears to be similar to the closed-loop incompatibility of adjacent steady states discussed above for input multiplicity.

Examination of Eq. 19 shows that a likely circumstance corresponding to an infinite value of dc/dm is a singularity of $\partial g/\partial x$ at the steady state. This is precisely what occurs with the exothermic CSTR. A singularity of $\partial g/\partial x$ implies a zero-eigenvalue for the dynamic system. A steady-state with a zero eigenvalue is clearly transitional between stability and instability. Thus it is tempting to conclude that the steady-state characteristic in Figure 2 imposes restrictions on dynamic open-loop stability of adjacent steady states similar to those imposed by the characteristics in Figure 1 on the dynamic closed-loop stability.

However, Eq. 19 shows that other circumstances can also cause an infinite value of dc/dm. As an example, consider a dynamic system described by

$$\frac{dx}{dt} = \frac{u - q(x)}{q'(x)} \tag{33}$$

$$y = x \tag{34}$$

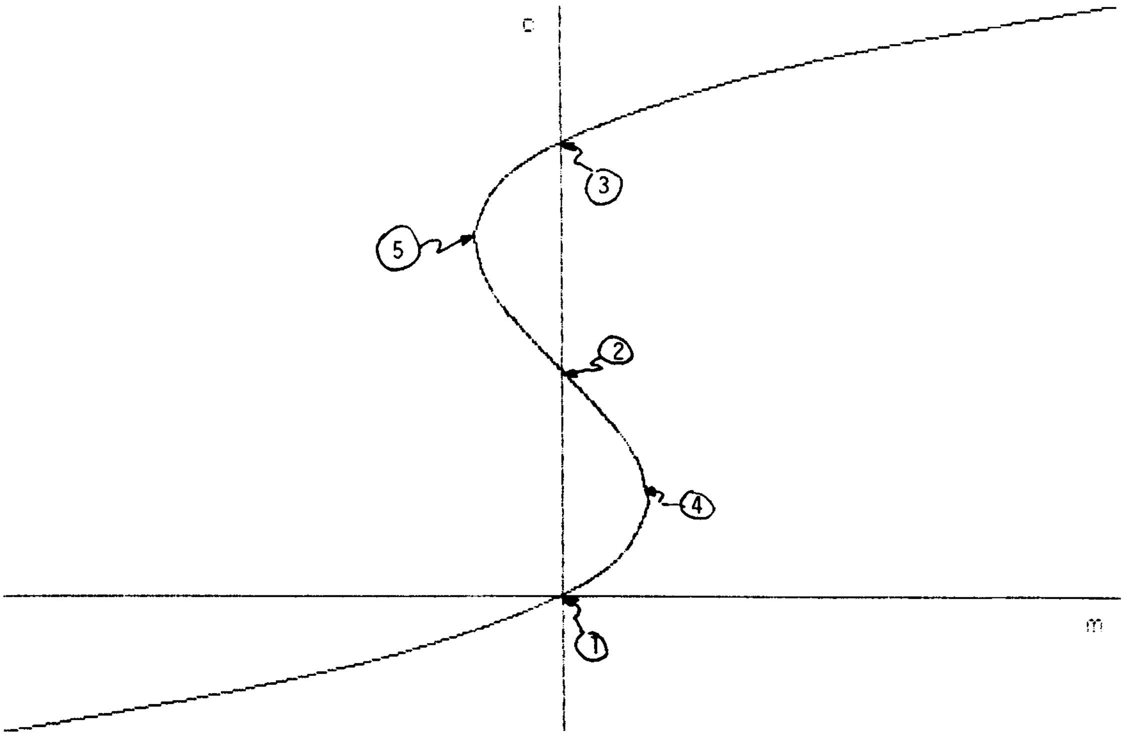

Figure 2. Steady-state characteristics of a process with output multiplicity.

where $q(x)$ is a function similar to the steady-state characteristic illustrated by Figure 2,

$$q(c) = m \tag{35}$$

and $q'(x)$ is its first derivative. For this case,

$$\frac{\partial g}{\partial x} = \frac{-[q'(x)]^2 + [u - q(x)]q''(x)}{[q'(x)]^2}$$

at steady-state, so the system is always stable. Thus, the three steady-states indicated as 1, 2 and 3, will all be stable. Here, the infinite value of dc/dm is caused by a zero of $q'(x)$, which results in an infinite value of $\partial g/\partial u$.

Obviously, there are both physical and mathematical difficulties with Eq. 33 as a dynamic system, because the time derivative is not adequately defined at the zeros of $q'(x)$. This could be remedied in a variety of ways. The point is that more details would be necessary to generalize the precise restrictions placed by the static characteristic on the dynamic stability in this case.

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NOTATION

A,B,C,D,E = intermediate matrices used in computations
c = vector of n process outputs c_i
e = vector of n errors or discrepancies
f = vector of n arbitrary functions of $n + p$ variables
g = vector of n arbitrary functions of $n + p$ variables
h = vector of n feedback relations
I = unit matrix
K_R = diagonal $(n \times n)$ matrix of reset gains k_{Ri}
m = vector of n manipulated inputs m_j
n = number of inputs and outputs
p = number of state variables
q = function with multiple roots, as illustrated in Figure 2
r = vector of n set points
s = Laplace transform variable
t = time
u,U = vector of n time-varying process inputs; and, vector of corresponding deviations
x,X = vectors of p state variables, and deviations
y,Y = vectors of n time-varying process outputs, and deviations
z,Z = vectors of n augmenting state variables, and deviations
μ = $(n \times n)$ square matrix of relative gains, with elements μ_{ij}
$+$ = superscript denoting sign normalization
s = subscript denoting steady state

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